

# Possible dark energy imprints in gravitational wave spectrum of mixed neutron-dark-energy stars

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## Abstract

In the present paper we study the oscillation spectrum of neutron stars containing both ordinary matter and dark energy in different proportions. Within the model we consider, the equilibrium configurations are numerically constructed and the results show that the properties of the mixed neutron-dark-energy star can differ significantly when the amount of dark energy in the stars is varied. The oscillations of the mixed neutron-dark-energy stars are studied in the Cowling approximation. As a result we find that the frequencies of the fundamental mode and the higher overtones are strongly affected by the dark energy content. This can be used in the future to detect the presence of dark energy in the neutron stars and to constrain the dark-energy models.

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# 1 Introduction

The cosmological observations of the present Universe provide evidences for the existence of a mysterious kind of matter, called dark energy, which governs the accelerated expansion of the Universe [1], [2]. The dark energy constitutes 73% of total energy content of the Universe and it exhibits some unusual properties such as negative pressure to density ratio  $w$  (in hydrodynamical language).

The fundamental role that the dark energy plays in cosmology naturally makes us search for local astrophysical manifestation of it. If dark energy was discovered at astrophysical scales it would give us new tools for more profound investigation of it and its properties. For example, the existence of dark energy makes us expect that some of the present-day stars are a mixture of both ordinary matter and dark energy in different proportions. The study of such mixed objects is a new interesting problem in the current investigations, see for example [3]– [11] and references therein.

The natural question that arises is how we can detect the presence of dark energy in the stars. Are there any imprints of dark energy that can be observed and how? In present paper we consider one such possibility based on the gravitational waves radiation by the neutron stars. It is well known that the characteristics, and more precisely the spectrum of the gravitational waves emitted by the compact objects, depend on the interior structure of the compact objects. This is the base of the asteroseismology – the interior structure of the compact objects can be revealed by investigating the spectrum of the gravitational waves they emit [12, 13]. Currently a lot of efforts are devoted to the detection of the gravitational waves. Several ground-based detectors as LIGO, VIGO, TAMA300 and GEO600 are operating and the launching of space-based detectors, for example LISA, are expected in near future. The detection of the gravitational waves will open a new window to the Universe and will provide us with a unprecedented tool for studying the compact astrophysical objects.

In the present paper we consider a certain model of mixed neutron-dark-energy stars (MNDES) and study their oscillation spectrum in the Cowling approximation. We show that the spectrum of MNDES contains imprints of dark energy and could be used to detect the presence of dark energy in the neutron stars.

## 2 Equilibrium model for mixed neutron-dark-energy stars

The nature of the dark energy is a mystery at present and it is a great challenge to current physics to solve this problem. Nevertheless, in their attempts to understand and model the dark energy, theoretical physicists have adopted some effective descriptions of it. The most simple and direct description is by a perfect fluid with a negative pressure. The other effective descriptions of the dark energy depend on the value of  $w$ . If  $w > -1$  a possible description is provided by a scalar field with an appropriately chosen potential. In the case when  $w = -1$  the dark energy can be explained by the cosmological constant. If

$w < -1$  then a possible description of dark energy is provided by scalar fields with negative kinetic energy, the so-called phantom (ghost) scalars [14], [15]. The negative kinetic energy, however, leads to severe quantum instabilities<sup>†</sup> and this is a formidable challenge to the theory. However, there are claims that these instabilities can be avoided [16]. In general, the problem could be avoided if we consider the phantom scalars as an effective field theory resulting from some kind of fundamental theory with a positive energy [17], [18]. In this case the phantom scalar description of the dark energy is physically acceptable. In this context it is worth noting that the phantom-type fields arise in string theories and supergravity [21]–[24].

At present it is not clear whether  $w < -1$ ,  $w = -1$  or  $w > -1$ . It seems, however, that the current observational constraints favor  $w < -1$  [25]–[30]. That is why in the present work we adopt an effective description of the dark energy by a phantom scalar. Since a very little is known about the interaction of dark energy with the normal matter we consider a simple model with minimal interactions for the phantom field – we do not include terms describing non-minimal interaction between the phantom field and the normal matter in the field equations [11]. Under these conditions the Einstein equations in the presence of dark energy read [11]

$$\begin{aligned} R_{\mu\nu} &= 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}) - 2\partial_\mu\varphi\partial_\nu\varphi, \\ \nabla_\mu\nabla^\mu\varphi &= 4\pi\rho_D. \end{aligned} \tag{1}$$

Here  $T_{\mu\nu}$  is the energy-momentum tensor of the ordinary matter in the perfect fluid description with energy density  $\rho$  and pressure  $p$ :

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}. \tag{2}$$

For the ordinary matter we impose the natural conditions  $\rho \geq p \geq 0$ . The density of the dark energy sources is denoted by  $\rho_D$ . The dark energy sources will be called *dark charges*. When we consider the local manifestation of dark energy on astrophysical scales, i.e. scales much smaller than the cosmological scales, the phantom potential  $\mathcal{U}(\varphi)$  can be neglected and that is why we have set  $\mathcal{U}(\varphi) = 0$ .

The contracted Bianchi identity applied to the field equations gives

$$\nabla_\nu T_\mu^\nu = \rho_D \nabla_\mu \varphi. \tag{3}$$

This equation projected orthogonally to the 4-velocity  $u^\mu$  gives the equation of motion of the fluid. In the case of static configurations (not necessary spherically symmetric) it reduces to the equation describing the hydrostatic equilibrium:

$$(\rho + p)\partial_i \ln \sqrt{|g_{tt}|} + \partial_i p = \rho_D \partial_i \varphi, \tag{4}$$

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<sup>†</sup>The perfect fluid description of the dark energy also suffers from instabilities due to the imaginary velocity of the sound. From a classical point of view the massless phantom field is even more stable than its usual counterpart [19], [20].

where the subscript  $i$  is for the spatial coordinates. For barotropic equation of state (EOS) of the ordinary matter  $p = p(\rho)$ , eq. (3) can be rewritten in the form

$$d \ln \sqrt{|g_{tt}|} + d \int \frac{dp}{\rho + p} = \frac{\rho_D}{\rho + p} d\varphi. \quad (5)$$

The integrability condition for this equation gives

$$\rho_D = f(\varphi)(\rho + p), \quad (6)$$

where  $f(\varphi)$  is arbitrary function of  $\varphi$ . In the present paper we consider the simplest choice for  $f(\varphi)$ , namely  $f(\varphi) = k$  where  $k \geq 0$  is a constant, i. e.

$$\rho_D = k(\rho + p). \quad (7)$$

In fact the constant  $k$  is a measure for the content of dark energy in the star, while  $(\rho + p)$  determines the dark energy profile in the star.

Now let us consider the projection of (3) along the 4-velocity  $u^\mu$ . We find

$$-\frac{d\rho}{d\tau} - (\rho + p)\nabla_\nu u^\nu = \rho_D \frac{d\varphi}{d\tau}, \quad (8)$$

where  $\frac{d}{d\tau}$  is the derivative along the fluid 4-velocity. Combining this equation with the baryon conservation law  $\nabla_\nu (nu^\nu) = 0$  we find

$$-\frac{d\rho}{d\tau} + \frac{(\rho + p)}{n} \frac{dn}{d\tau} = \rho_D \frac{d\varphi}{d\tau}, \quad (9)$$

or equivalently

$$d\rho = \frac{(\rho + p)}{n} dn - \rho_D d\varphi, \quad (10)$$

which is the thermodynamics first law for our system of ordinary matter and dark energy. Hence we conclude that

$$\rho = \rho(n, \varphi), \quad (11)$$

i.e. the energy density is a function not only of the particle number density but depends also on the scalar field. In order to find the function  $\rho(n, \varphi)$  we use the barotropic equation  $p = p(\rho)$  and eq. (7) which substituted in (10) and some algebra gives

$$\int_0^\rho \frac{d\rho}{\rho + p} = \int_0^{n_*} \frac{dn_*}{n_*}, \quad (12)$$

where

$$n_* = e^{-k\varphi} n. \quad (13)$$

Eq. (12) determines the function  $\rho(n, \varphi)$  and it is easy to see that if in the absence of dark energy  $\rho(n) = F(n)$ , then  $\rho(n, \varphi) = F(n_*) = F(e^{-k\varphi} n)$  in the presence of dark energy.

From now on we will focus on spherically symmetric equilibrium solutions. In this case the spacetime metric can be written in the well-known form

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (14)$$

and the reduced field equations are the following

$$\begin{aligned} \frac{d}{dr} [r(1 - e^{-2\Lambda})] &= 8\pi\rho r^2 - r^2 \left( \frac{d\varphi}{dr} \right)^2 e^{-2\Lambda}, \\ \frac{d\Phi}{dr} &= \frac{1}{2r} \left[ e^{2\Lambda} - 1 + 8\pi p r^2 e^{2\Lambda} - r^2 \left( \frac{d\varphi}{dr} \right)^2 \right], \\ \frac{d}{dr} \left[ e^{\Phi-\Lambda} r^2 \frac{d\varphi}{dr} \right] &= 4\pi\rho_D r^2 e^{\Phi+\Lambda}, \end{aligned} \quad (15)$$

together with the equation describing hydrostatic equilibrium (4) in the spherically symmetric case

$$(\rho + p) \frac{d\Phi}{dr} + \frac{dp}{dr} = \rho_D \frac{d\varphi}{dr}. \quad (16)$$

Here it is worth noting that the system of equations (15)–(16), can be viewed as describing an equilibrium spherically symmetric configuration with an effective anisotropic pressure<sup>‡</sup>. As a hole we can say that the presence of dark energy in the star yields a pressure anisotropy of certain kind and (7) can be considered as an effective quasi-local equation of state for the anisotropic part of the pressure. The local anisotropy also affects the thermodynamics first law as we have already seen in our case.

We numerically solve the system (15)–(16) with the following boundary conditions

$$\Lambda(0) = 0, \quad \rho(0) = \rho_0, \quad \frac{d\varphi}{dr}(0) = 0, \quad \varphi(\infty) = 0, \quad \Phi(\infty) = 0. \quad (17)$$

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<sup>‡</sup> For a discussion of locally anisotropic selfgravitating systems we refer the reader to [31].

The radius  $R$  of the star is determined from the condition  $p(R) = 0$ . We also use a polytropic equations of state for the ordinary matter

$$p = p(n_*) = K n_0 m_b \left( \frac{n_*}{n_0} \right)^\Gamma = K n_0 m_b \left( e^{-k\varphi} \frac{n}{n_0} \right)^\Gamma, \quad (18)$$

$$\rho = \rho(n_*) = n_* m_b + \frac{p}{\Gamma - 1} = e^{-k\varphi} n m_b + \frac{p}{\Gamma - 1}, \quad (19)$$

where  $n$  is the particle number density,  $m_b = 1.66 \times 10^{-24} g$ ,  $n_0 = 0.1 fm^{-3}$ . We have chosen  $\Gamma = 2.46$  and  $K = 0.00936$  obtained when fitting the tabulated data for EOS II [32], but the results are qualitatively the same for other values  $K$  and  $\Gamma$ .

The mass, the number of baryons and the dark charge of the star are given by [11]

$$M = -\frac{1}{4\pi} \int_{Star} R_t^t \sqrt{-g} d^3x = \int_{Star} (\rho + 3p) \sqrt{-g} d^3x = 4\pi \int_0^R (\rho + 3p) e^{\Phi+\Lambda} r^2 dr, \quad (20)$$

$$N = 4\pi \int_0^R n e^\Lambda r^2 dr, \quad (21)$$

$$D = \int_{Star} \rho_D \sqrt{-g} d^3x = 4\pi \int_0^R \rho_D e^{\Phi+\Lambda} r^2 dr = \frac{1}{4\pi} \oint_{S_\infty^2} \nabla_\mu \varphi d\Sigma^\mu. \quad (22)$$

Other equivalent formulas for calculating  $M$  and  $D$  can be obtained by using eqs. (15).

The numerical results for the background solutions are shown on Fig. 1 where the mass  $M$  as a function of the central density  $\rho_0$  and of the radius  $R$  are presented. On Fig. 2 the dark charge  $D$  and the ratio of the dark charge to the total mass  $D/M$  of the MNDES as functions of the central density are shown. The sequences of solutions are obtained when varying the value of the central density of the ordinary matter  $\rho_0$  while keeping the dark energy parameter  $k$  fixed. As we can see the properties of the star can vary significantly when the portion of the dark energy inside the star increases, i.e. when we increase  $k$ . For large values of  $k$  (i.e. large portions of dark energy) the total mass of the star can be much larger than the pure neutron star mass for the same equation of state and the radius of the star also increases. These effects are due to the phantom field which yields repulsive rather than attractive force. In fact the additional force on the right hand side of eq.(16) is repulsive,  $\rho_D \frac{d\varphi}{dr} > 0$  and this can be easily seen from the field equation for  $\varphi$ . In hydrodynamical language, the additional force due to the anisotropic nature of the fluid is pointing outward. We have found numerically that there are no equilibrium configurations for  $k \gtrsim 1$  since the repulsive force dominates on the attractive gravity in that case.

Now let us turn to the question about the stability of the equilibrium configurations. The dynamical stability is usually studied by considering their radial perturbations and reducing the problem to an eigenvalue problem. Since the equilibrium configurations in our case are parameterized by one parameter (for fixed  $k$ ), say the central density of the ordinary matter  $\rho_0$ , the eigenvalues will be continuous functions of them. What is important in the present

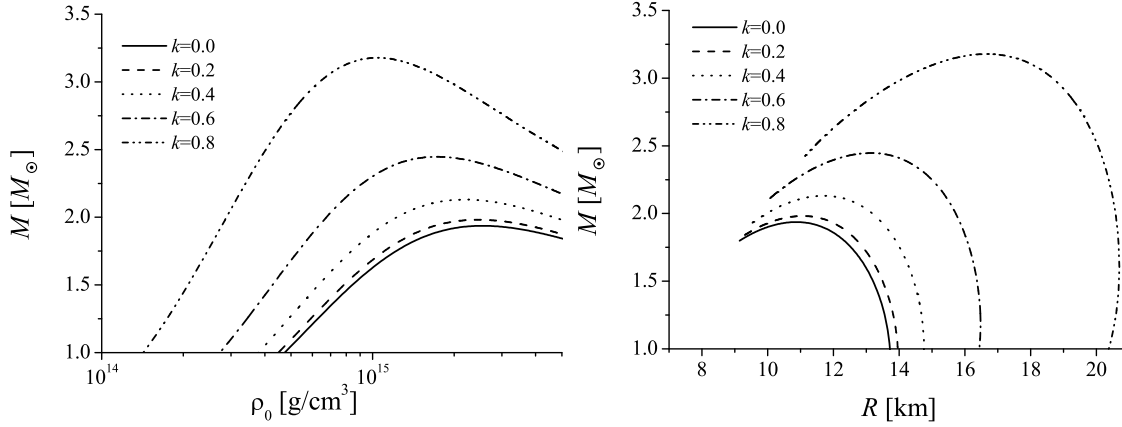


Figure 1: The mass of the MNDES as a function of the central density (left panel) and of radius of the ordinary matter component (right panel) for several values of the dark energy parameter  $k$  ( $k = 0$  corresponds to a pure neutron star).

context is the point between the stable and instable configurations. On that point we must have a zero eigenvalue or in other words a static perturbation transforming the equilibrium state with  $\rho_0$  to the equilibrium state with  $\rho_0 + \delta\rho_0$ . The static perturbation preserves the number of particles  $N$  of the ordinary matter. Therefore the perturbed equilibrium state have the same  $N$  as the initial equilibrium state which means that

$$\delta N = \frac{\partial N}{\partial \rho_0} \delta \rho_0 = 0. \quad (23)$$

Therefore the change of stability is where  $\frac{\partial N}{\partial \rho_0} = 0$ . Moreover, for our model and for fixed  $k$ , one can show that<sup>§</sup>

$$\delta M = \tilde{\mu} \delta N, \quad (24)$$

where  $\tilde{\mu}$  is the red-shifted chemical potential,  $\tilde{\mu} = \frac{\rho+p}{n} e^\Phi$ . It can be shown from (16) that  $\tilde{\mu}$  is constant throughout the star. Consequently the mass and the particle number peak at the same location, as one can see from Fig. 3. Therefore the stability change occurs at the critical energy density  $\rho_0^c$  for which  $\frac{\partial M}{\partial \rho_0} = 0$ . The configurations with  $\rho_0 < \rho_0^c$  are stable while those with  $\rho_0 > \rho_0^c$  are instable because the configurations with  $\rho_0 < \rho_0^c$  have lower energy than those with  $\rho_0 > \rho_0^c$  as one sees on Fig. 3.

### 3 Perturbation equations in Cowling approximation

In this section we derive the equations describing the non-radial perturbations of the MNDES in the so-called Cowling approximation [33]– [34]. In Cowling approximation the

<sup>§</sup>More precisely we have  $\delta M = \tilde{\mu} \delta N + D \delta \varphi_\infty$ , however we consider fixed cosmological value  $\varphi_\infty$ .

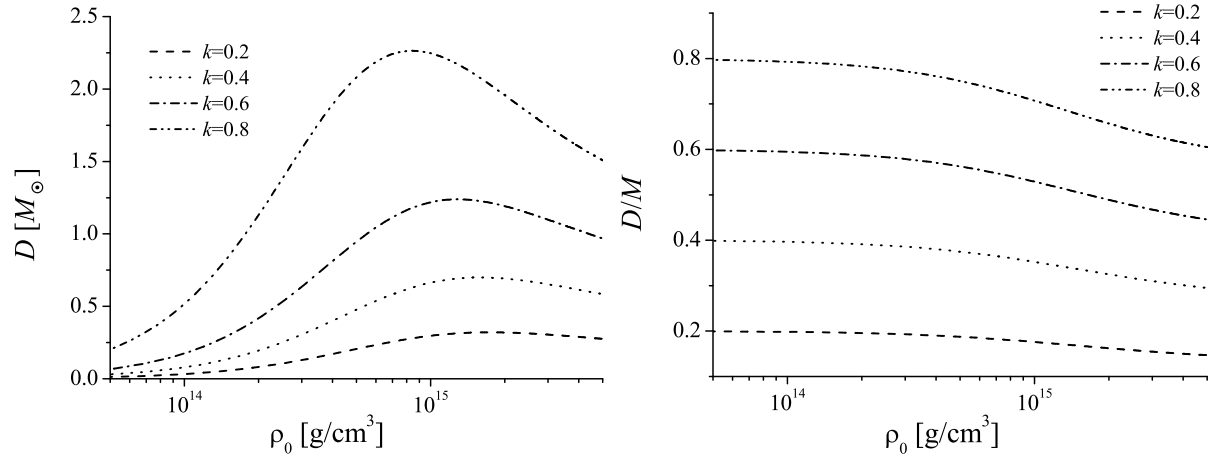


Figure 2: The dark charge in solar mass units (left panel) and the ratio of the dark charge to the total mass of the MNDES (right panel) as functions of the central density. The results are for the same solutions as those shown on Fig. 1.

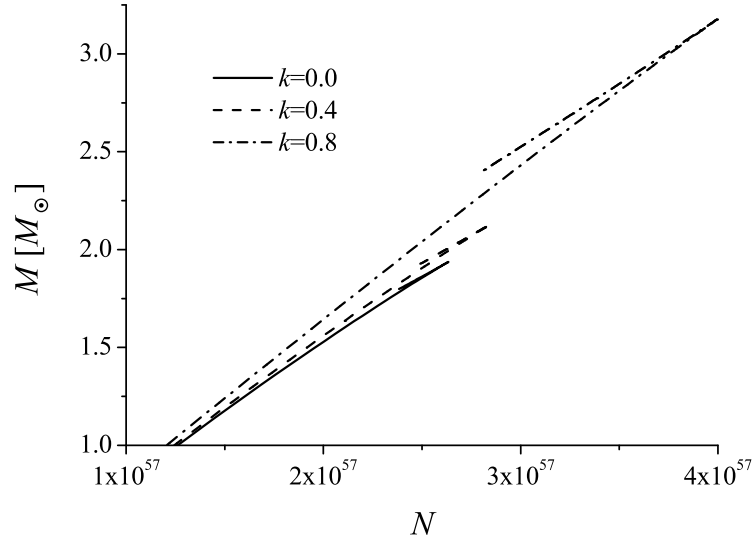


Figure 3: The dependence of the star mass on the ordinary matter particle number for different values of  $k$ .



non-fluid degrees of freedom are kept fixed – for example, within the framework of general relativity, the spacetime metric is kept fixed. Despite of this simplification the Cowling formalism turns out to be accurate enough and reproduces the oscillation spectrum with good accuracy. In fact the comparison of the oscillation frequencies obtained by a fully general relativistic numerical approach and by the Cowling approximation shows that the discrepancy is less than 20% for the typical stellar models [35]. The Cowling approximation is trustable even in the case of presence of scalar fields [36]. We apply the Cowling approximation to our model by keeping the metric and the scalar phantom field frozen.

The equations describing the perturbations in Cowling formalism are obtained by varying the equation for the conservation of the energy-momentum tensor (3). Taking into account that the metric and scalar field are frozen, we find  $\nabla_\nu \delta T_\mu^\nu = 0$  where

$$\delta T_\mu^\nu = (\delta\rho + \delta p) u_\mu u^\nu + (\rho + p) (u_\mu \delta u^\nu + \delta u_\mu u^\nu). \quad (25)$$

Projecting the equation  $\nabla_\nu \delta T_\mu^\nu = 0$  along the background 4-velocity  $u^\mu$  we have

$$u^\nu \nabla_\nu \delta\rho + \nabla_\nu [(\rho + p) \delta u^\nu] + (\rho + p) a_\nu \delta u^\nu = 0. \quad (26)$$

Projecting orthogonally to the background 4-velocity by using the operator  $P_\mu^\nu = \delta_\mu^\nu + u^\nu u_\mu$ , we obtain

$$(\delta\rho + \delta p) a_\mu + (\rho + p) u^\nu (\nabla_\nu \delta u_\mu - \nabla_\mu \delta u_\nu) + \nabla_\mu \delta p + u_\mu u^\nu \nabla_\nu \delta p = 0, \quad (27)$$

where  $a_\mu = u^\nu \nabla_\nu u_\mu$  is the background 4-acceleration. Taking into account that  $u = (u^t, 0, 0, 0)$  the above equation can be rewritten in the form

$$(\delta\rho + \delta p) a_i + (\rho + p) u^t \partial_t \delta u_i + \partial_i \delta p = 0 \quad (28)$$

with  $i = 1, 2, 3 = r, \theta, \phi$

At this stage we can express the perturbations of the 4-velocity via the Lagrangian displacement vector  $\xi^i$ , namely

$$\frac{\partial \xi^i}{\partial t} = \frac{\delta u^i}{u^t}. \quad (29)$$

Now let us consider eq.(28) for  $i = 1$  and  $i = 2$ . Since  $a_\theta = a_\phi = 0$  we find

$$(\rho + p)(u^t)^2 \partial_t^2 \xi_\theta + \partial_\theta \delta p = 0, \quad (30)$$

$$(\rho + p)(u^t)^2 \partial_t^2 \xi_\phi + \partial_\phi \delta p = 0. \quad (31)$$

Taking into account that  $\rho, p$  and  $u^t$  depend on  $r$  only, the integrability condition for the above equations gives

$$\partial_\theta \xi_\phi = \partial_\phi \xi_\theta. \quad (32)$$

From this condition and the fact that the background is spherically symmetric we find that  $\xi_\theta$  and  $\xi_\phi$  are of the form

$$\xi_\theta = - \sum_{lm} V_{lm}(r, t) \partial_\theta Y_{lm}(\theta, \phi), \quad (33)$$

$$\xi_\phi = - \sum_{lm} V_{lm}(r, t) \partial_\phi Y_{lm}(\theta, \phi), \quad (34)$$

where  $Y_{lm}(\theta, \phi)$  are the spherical harmonics. From now on, in order to simplify the notations, we will just write  $\xi = -VY_{lm}$  when we have expansion in spherical harmonics.

We proceed further with finding the expressions for the density and pressure perturbations. From eq.(27) we have

$$u^t \delta \rho = - \frac{1}{\sqrt{-g}} \partial_i [\sqrt{-g}(\rho + p) u^t \delta \xi^i] - (\rho + p) a_i \xi^i. \quad (35)$$

It is convenient to express  $\xi^r$  in the form

$$\xi^r = e^{-\Lambda} \frac{W}{r^2} Y_{lm} \quad (36)$$

and substituting in the above equations, after some algebra we find<sup>¶</sup>

$$\delta \rho = -(\rho + p) \left[ e^{-\Lambda} \frac{W'}{r^2} + \frac{l(l+1)}{r^2} V \right] Y_{lm} - \left( \frac{d\rho}{dr} + \rho_D \frac{d\varphi}{dr} \right) e^{-\Lambda} \frac{W}{r^2} Y_{lm}, \quad (37)$$

where in the last step we have taken into account that  $a_r = \Phi'$  and eq. (16).

In order to find the perturbation of the pressure we first use the relation between the Eulerian and Lagrangian variations, namely

$$\delta p = \Delta p - \xi^r \partial_r p \quad (38)$$

with  $\Delta p$  being the Lagrangian variation. From the equation of state we have

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<sup>¶</sup>The derivative with respect to the radial coordinate  $r$  will be denoted by prime or by the standard symbol interchangeably.

$$\Delta p = \frac{dp}{d\rho} \Delta \rho = \frac{dp}{d\rho} (\delta \rho + \xi^r \partial_r \rho). \quad (39)$$

In this way we obtain the following formula for the perturbation of pressure

$$\delta p = \frac{dp}{d\rho} \left\{ (\rho + p) \left[ e^{-\Lambda} \frac{W'}{r^2} + \frac{l(l+1)}{r^2} V \right] + \rho_D \frac{d\varphi}{dr} e^{-\Lambda} \frac{W}{r^2} \right\} Y_{lm} - \frac{dp}{dr} e^{-\Lambda} \frac{W}{r^2} Y_{lm}. \quad (40)$$

Having the explicit expressions for  $\delta \rho$  and  $\delta p$  we can put them into eq.(28) and after lengthy calculations we obtain the dynamical equations for  $W$  and  $V$

$$\begin{aligned} (\rho + p) \frac{e^{\Lambda-2\Phi}}{r^2} \partial_t^2 W &= \frac{d}{dr} \left\{ (\rho + p) \frac{dp}{d\rho} \left[ e^{-\Lambda} \frac{W'}{r^2} + \frac{l(l+1)}{r^2} V \right] + \frac{dp}{dr} e^{-\Lambda} \frac{W}{r^2} + \frac{dp}{d\rho} \rho_D \frac{d\varphi}{dr} e^{-\Lambda} \frac{W}{r^2} \right\} \\ &- \Phi' \left\{ (\rho + p) \left( 1 + \frac{dp}{d\rho} \right) \left[ e^{-\Lambda} \frac{W'}{r^2} + \frac{l(l+1)}{r^2} V \right] \right. \\ &+ \left. \left( \frac{d\rho}{dr} + \frac{dp}{dr} \right) e^{-\Lambda} \frac{W}{r^2} + \left( 1 + \frac{dp}{d\rho} \right) \rho_D \frac{d\varphi}{dr} e^{-\Lambda} \frac{W}{r^2} \right\}, \end{aligned} \quad (41)$$

$$(\rho + p) e^{-2\Phi} \partial_t^2 V = (\rho + p) \frac{dp}{d\rho} \left[ e^{-\Lambda} \frac{W'}{r^2} + \frac{l(l+1)}{r^2} V \right] + \frac{dp}{dr} e^{-\Lambda} \frac{W}{r^2} + \frac{dp}{d\rho} \rho_D \frac{d\varphi}{dr} e^{-\Lambda} \frac{W}{r^2}. \quad (42)$$

From now on we will assume for the perturbation functions a harmonic dependence on time, i.e.  $W(r, t) = W(r) e^{i\omega t}$  and  $V(r, t) = V(r) e^{i\omega t}$ . The system (41)–(42) can be considerably simplified by combining the equations in an appropriate manner. When differentiating equation (42) and adding it to equation (41), and also using eq.(16), we find

$$V' = 2\Phi' V - e^\Lambda \frac{W}{r^2} - \frac{1 + \frac{d\rho}{dp}}{\rho + p} \rho_D \frac{d\varphi}{dr} V. \quad (43)$$

This equation together with the equation (42) solved for  $W'$ , form a system which is equivalent to (41)–(42) but much more tractable:

$$W' = \frac{d\rho}{dp} \left[ \omega^2 r^2 e^{\Lambda-2\Phi} V + \Phi' W \right] - l(l+1) e^\Lambda V - \frac{1 + \frac{d\rho}{dp}}{\rho + p} \rho_D \frac{d\varphi}{dr} W, \quad (44)$$

$$V' = 2\Phi' V - e^\Lambda \frac{W}{r^2} - \frac{1 + \frac{d\rho}{dp}}{\rho + p} \rho_D \frac{d\varphi}{dr} V.$$

The boundary condition at the star surface is that the Lagrangian perturbation of the pressure vanishes

$$(\rho + p) \left( \omega^2 e^{-2\Phi} V + \Phi' e^{-\Lambda} \frac{W}{r^2} - \frac{\rho_D}{\rho + p} \frac{d\varphi}{dr} e^{-\Lambda} \frac{W}{r^2} \right) = 0. \quad (45)$$

The boundary conditions at the star center can be obtained by examining the behaviour in the vicinity of  $r = 0$ . For this purpose it is convenient to introduce the new functions  $\tilde{W}$  and  $\tilde{V}$  defined by

$$W = \tilde{W} r^{l+1}, \quad V = \tilde{V} r^l. \quad (46)$$

Then one can show that at  $r = 0$  the following boundary condition is satisfied

$$\tilde{W} = -l\tilde{V}. \quad (47)$$

## 4 Numerical results for the oscillation spectrum

As we have shown in the previous section the properties of the MNDES depends strongly on the dark energy content of the star and the masses and the radii can differ significantly. It is natural to expect that this will influence the oscillations of the star. In order to find the oscillation spectrum of MNDES we have numerically solved the spectral problem given by the differential equations (44) together with the boundary conditions (45) and (47). In this section we will present the results for the  $f$ -modes of the MNDES and for the higher fluid modes –  $p_1$  and  $p_2$ . All the dependencies are shown up to the maximum mass for the corresponding values of the parameters, i.e. where a change of stability is observed.

For pure neutron star an empirical relation between the average density and the  $f$ -mode frequency exists<sup>||</sup> which does not depend much on the EOS [12]. As we can see from Fig. 4 this relation differ for MNDES with different dark energy content (i.e. for different values of  $k$ ), but it remains relatively close to the pure neutron star (the case with  $k = 0$ ). Only for large values of  $k$  the relation changes more significantly – the frequencies grow slower as a function of the average density.

The influence of the dark energy on the oscillation spectrum is more pronounced on the Figs. 5, 6 and 7 where it is shown the oscillation frequencies  $f$  and the normalized frequency  $\omega$  as a function of the total mass  $M$  of the MNDES for the  $f$ ,  $p_1$  and  $p_2$  modes. The values of the frequencies  $f$  decrease when we increase the dark energy content for all of the fluid modes and the differences with the pure neutron star case ( $k = 0$ ) can be large. This effect is more pronounced for the higher fluid modes and for larger values of  $k$  the frequencies of the  $p_2$ -mode can reach two times the frequencies of the pure neutron star case as we can see

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<sup>||</sup>The  $f$ -mode frequency is proportional to the average density.

on Fig. 7. This is a very strong effect and by observing more than one fluid mode of the same neutron star we can put serious constraints on the dark energy content in the MNDES and consequently on the dark energy model itself.

An interesting observation is that the normalized frequency  $\omega$  behaves differently as a function of the total mass  $M$  for the different fluid modes as we can see from the right panel on Figs. 5, 6 and 7. The results show that the normalized  $\omega$  increases when we increase the parameter  $k$  for the  $f$ -mode whereas for the higher fluid modes  $\omega$  decreases with the increase of the dark energy content. This effect is qualitatively different from most of the other alternative models of neutron stars \*\* where the qualitative behaviour of the normalized frequencies  $\omega$  is the same for all of the modes when we vary the parameters of the corresponding model. This property can be potentially used to distinguish between MNDES and other alternative neutron star model. Again the differences between oscillation frequencies of the pure neutron star and of the MNDES are larger for the higher order modes.

These properties of the oscillation spectrum can be easily seen also on Fig. 8 where the normalized frequencies of the  $f$ ,  $p_1$  and  $p_2$ -modes are shown as a function of  $k$  for fixed value of the MNDES mass  $M = 1.4M_\odot$ . The results show that when we increase the value of  $k$  not only the absolute values of the frequencies change in a bigger range for the higher order fluid modes, but also the relative change in the frequencies is larger. On Fig. 8 we can also observe one of the properties of the oscillation spectrum that we discussed above – the normalized frequency increases when we increase the parameter  $k$  for the  $f$ -mode whereas for the higher fluid modes  $\omega$  decreases with the increase of  $k$ .

## 5 Conclusions

In the present paper a model of neutron star containing dark energy and its oscillation spectrum are studied. We model the dark energy part of the star by using a phantom scalar field [14], [15], [11]. The neutron-dark-energy star solutions are obtained by solving numerically the reduced system of field equations. The results show that the properties of the MNDES can be significantly different from the pure neutron star case. One of the main difference is that the masses and the radii can be significantly larger in the presence of dark energy. The stability of the MNDES is also studied and it turns out that similar to the pure neutron star case a change of stability is observed at the critical energy density  $\rho_0^c$  for which  $\frac{\partial M}{\partial \rho_0} = 0$ .

The second part of the paper is concentrated on examining the possible dark energy imprints in gravitational wave spectrum of the obtained MNDES. In order to do that we study the oscillation spectrum of the obtained solutions in the Cowling approximation where the non-fluid degrees of freedom are kept fixed. The values of the oscillation frequencies of the MNDES can differ significantly from the pure neutron star case and the differences are bigger for the higher fluid modes. Thus the observation of gravitational waves emitted from

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\*\*Such alternative models are for example the neutron stars in the scalar-tensor theories [36] or in the tensor-vector-scalar theory of gravity [37].

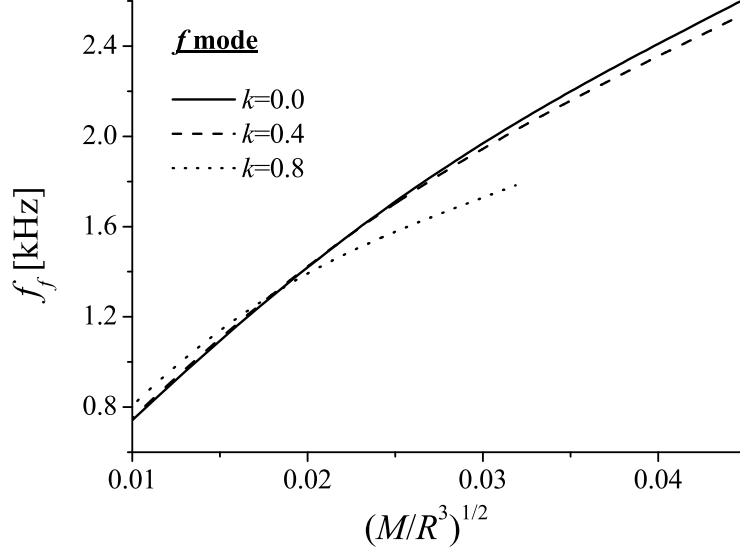


Figure 4: The  $f$ -mode frequency as a function of the average density of the ordinary matter component  $\sqrt{M/R^3}$ . The results for several values of the dark energy parameter  $k$  are shown.

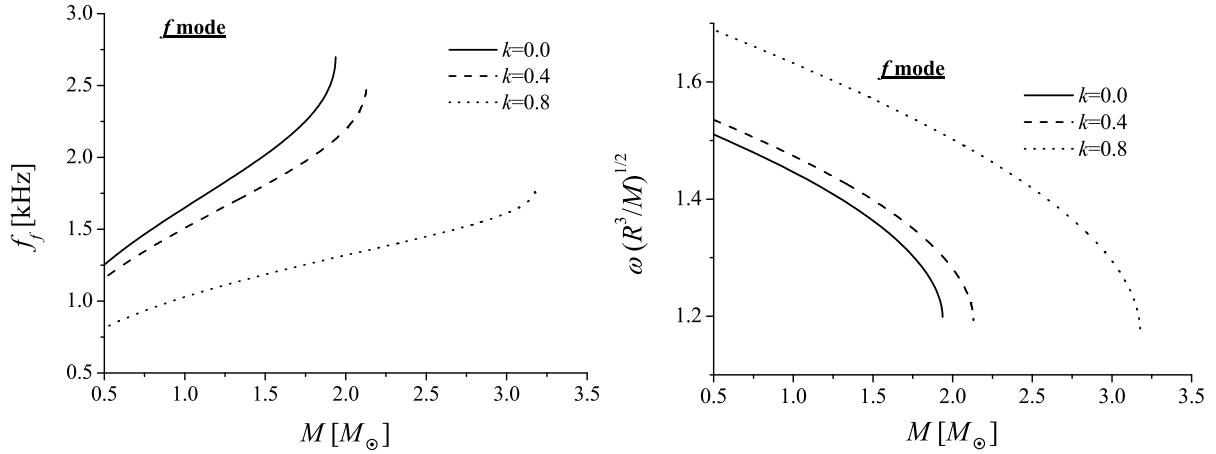


Figure 5: The frequency  $f$  as a function of the mass  $M$  (left panel) and the normalized frequency  $\omega$  as a function of  $M$  (right panel) for the  $f$  mode. The results for several values of the dark energy parameter  $k$  are shown.

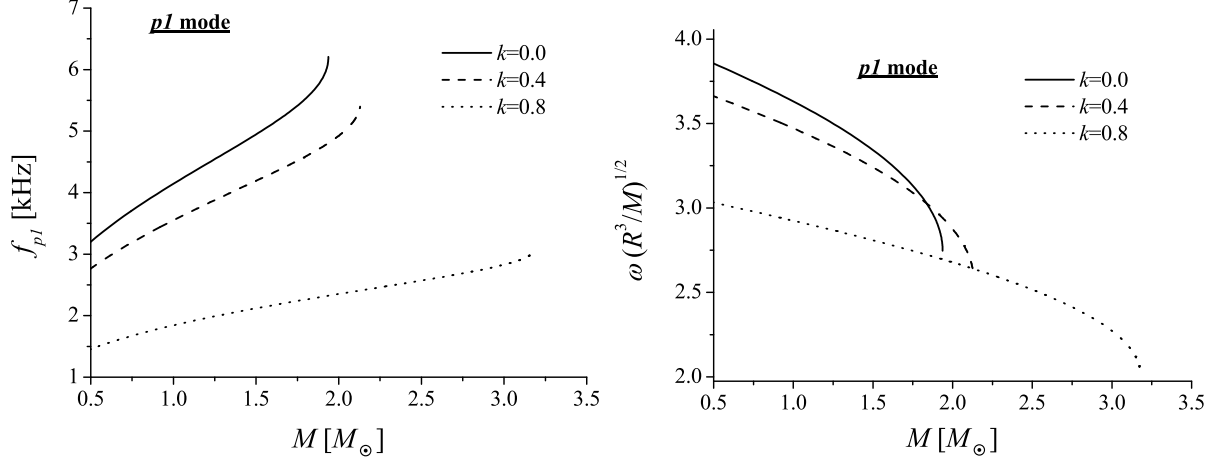


Figure 6: The results for the  $p_1$ -mode of the same solutions as shown on Fig. 5.

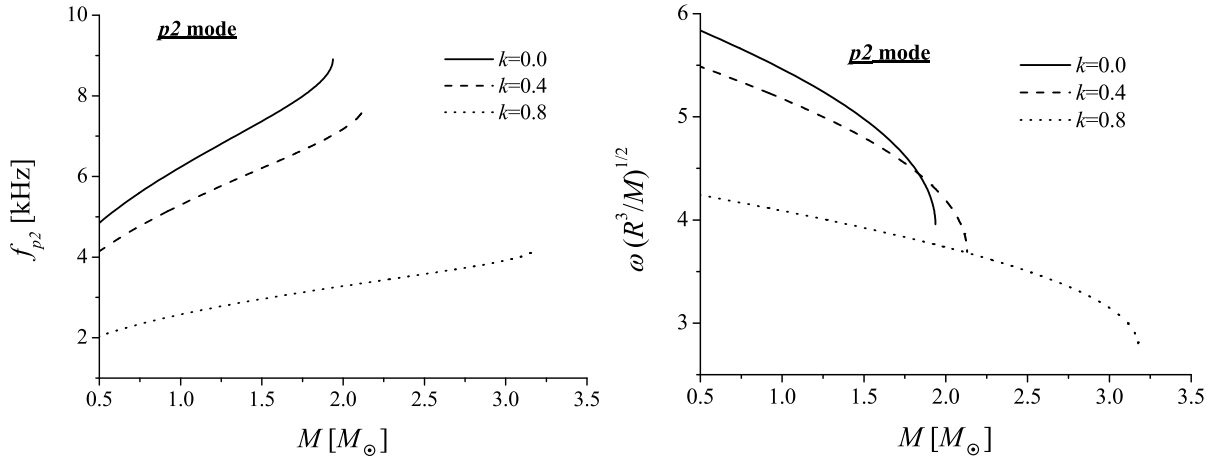


Figure 7: The results for the  $p_2$ -mode of the same solutions as shown on Fig. 5.

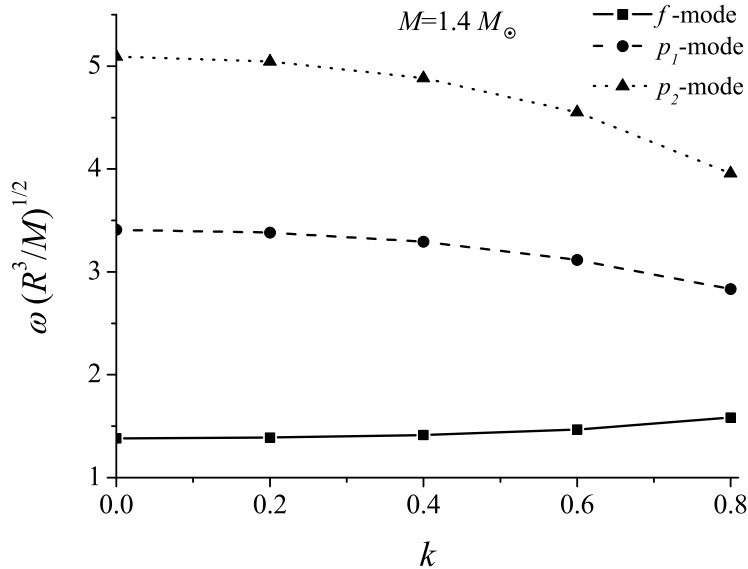


Figure 8: The normalized frequency  $\omega$  as a function of  $k$  for fixed value of the mass  $M = 1.4M_{\odot}$ . The results for the  $f$ ,  $p_1$  and  $p_2$  modes are shown.

neutron stars could be used to detect the presence of dark energy in the neutron stars and to put constraints on the dark energy properties in general.

We will finish by mentioning a possible further extension of the present work. Since the scalar phantom field violates the weak/null energy condition it can maintain configurations with nontrivial topology – the so-called wormholes. Such a solution describing a star harbouring a wormhole at its core was recently found in [10]. It would be very interesting to study the influence of the possible nontrivial topological structures in neutron stars, like wormholes, on the gravitational wave spectrum of the stars.

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